EVOLVING STREET PLANS, FROM SHOPS AND SHOPPERS

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Abstract

We report on research in which urban growth processes are simulated using software agents with vision of their environment as that is constrained by the location of buildings. The agents are free to build buildings (which then become shops), while other agents search for shops to satisfy a need for specific categories of goods. In effect these simulations create an artificial economy, and yet it is one in which the economic agents have forward facing vision, and in which their actions in locating buildings affect the visual field afforded to all other agents. The aim is to understand how an 'embodied and embedded' form of agency relates to an economy. In particular we are interested in the emergence of recognisable urban morphologies and distributions of different retail categories. Results are reported which show that linear streets emerge from this process, with under certain conditions these forming deformed grid like structures. However we find that minor variations in parameters have radical effects on the diversity of retail categories as well as on the emergent morphology. In particular we find that where agents do not have knowledge of store location more street like urban forms evolve, however where tax is high it forces layouts that minimise agent travel distances. We conclude with a discussion of the implications of these findings for our understanding of urban economic processes.

Introduction

In recent years space syntax research has added a series of explanatory layers to our understanding of urban spatial morphology and its effects. First, the insight that a fundamental result of spatial pattern was a corresponding pattern of movement (Hillier et al. 1993); next, that the by-product of movement generated a 'movement economy' in which passing trade produced by spatial configuration led to location of land uses and development densities (Hillier & Penn, 1992). Empirical research into retail space has found distinctly different patterns of agglomeration amongst comparison stores and dispersion for convenience stores, and hypothesised a coupling between spatial configuration and economic processes in order to explain this (Hossain, 2000; Hossain & Penn, 1999; Penn, 2005).

Further work has now added a cognitive mechanism to account for correlations between observed movement rates and spatial configuration (Penn, 2003; Hillier & Iida, 2005). These findings argue against the solely economic assumptions of urban geography: that movement is driven by the attractor properties of land uses and that decision behaviour is based on a rational assessment of travel costs in terms of distance or time. More recently, simulation experiments using software agents with vision of their environment have found that agent efficiency in finding different shops depends on both the global spatial structure of the urban grid and the degree of agglomeration or dispersion of shop premises (Penn & Turner, 2004). It seems clear that from an individual sighted agent's point of view both the morphology of the street pattern and the aggregation or dispersion of retail premises in that will have considerable effects upon their economic behaviour: that their behaviour as a rational agent is strongly constrained by the morphology and land use distribution of the world they inhabit and move through. What happens then if these factors – morphology and land use distribution – also become subject to dynamic economic processes dependent upon the population of mobile sighted agents? Turner has previously described this type of dynamic feedback between sighted agents and enviornments and conjectured that 'ecomorphic' environments can constitute a form of exosomatic learning (Turner, 2003). Here we test that conjecture also with regard to economic processes.

In order to investigate this question we have built an agent based simulation in which both sighted agents and the shop premises, which define the visual field open to an agent from any point of view, are dynamic. The sighted agents form a mobile population of 'shoppers' with shopping lists that they want to fulfil. From time to time a sighted agent will be converted into a shop owner, and will 'build' a shop within the open space of the city. Shops are square premises with a front face (indicated by a line on the map) which sell a single type of commodity (where 'type' is just a number and so might best be considered as its price), however shops are also economic entities. They have a starting balance, and gain income from any agent who stops to shop at them. For this to happen the commodity the shop sells must match an item on a shopper's list (within a defined tolerance). Different commodities are simply defined as numbers on a scale between 'cheap' (blue on the visualisation maps) and 'expensive' (red). The premises pay a tax or community charge as well as a rental for the site determined on the basis of the number of shoppers who pass by, and so the shopowner's balance decreases over time. It is therefore possible for a shop to go bankrupt if it fails to attract sufficient shoppers, in which case the premises are demolished and the agent is converted from a shopowner to a shopper once more.

Our research took the form of a series of experimental runs of the simulation in which we tested for the effects of different parameters or assumptions in the simulation. The key assumptions we tested related to the effects of level of taxation, the provision to the agent of perfect knowledge of the location of shops selling the goods they required (as opposed to no knowledge above that which could be directly seen from their current location and heading), and the provsion of the agent with multiple or single item shopping lists. As a simulation runs over time it can be considered as an optimisation process in which settlement morphology as well as the patterns of aggregation or dispersion of those selling diffent commodities evolves through the placement and demolition of shops in different locations. In each simulation the performance of agents in meeting their needs and the performance of shops in selling their wares were monitored, while the evolution of the physical morphology of the landscape and location of shops selling different categories of goods was visualised. In the subsequent sections we describe the simulations in greater detail before describing the main results for a series of experiments. Finally we discuss these with regard to their implications for our understanding of urban economic and spatial processes.

The Simulation

We shall be dealing with economics as far as it might influence the survival or otherwise of shops assuming a supply of customers with unlimited wealth, within different taxation regimes and with different ways of meeting their needs. The economic assumption, that the shop selling an item at the cheapest price always gets the most customers, will be compromised by a combination of lack of knowledge on the part of shoppers as to where they might find the best price and the extra cost of travelling to the location selling at the lowest price versus buying it at a more convenient location. We also have to look at how often the need for a particular object occurs to a shopper. 103-03

We can simply provide shoppers with the ‘knowledge’ they need by giving them perfect knowledge of the location of the shops (a map coordinate), and at the same time we can set values for the cost of travel. From the resultant matrix of values we hope to find anomalies which might be due to details of the configuration of the city, the directness of route, the lack of crowds or other factors, such as a diversity of shops which would make one route preferable to another.

The Shops

The shops are given an initial starting balance set using a dialogue. If the balance goes to zero the shop becomes a shopper. The balance will be incremented by the cost of the item the shop sells when a shopper with a need which matches the commodity offered by the shop approaches within a set distance. A shop only sells one kind of item. The balance can be decremented in several ways:

a) If a shopper approaches from behind a shop and a ‘thief tax’ has been set, the amount is decremented by the cost of items sold multiplied by the tax set. This tends to add costs to isolated buildings, which in the real world would have costs associated with building and heating, so the name “thief tax” is possibly a misnomer;

b) We have a facility to apply costs to shop owners at each iteration. There is a flat rate “tax” which can be applied to every shop. There is also a cost which can be applied dependent on the number of shoppers passing in front of the shop. In the results this is described as “rent”. We can also set both the above to increase or decrease dependent on either the average shop life or the proportion of shops to shoppers in the system as a whole. These modes are described as “dynamic tax” and “dynamic rent” in the results;

c) If a shopper finds itself in close proximity to a shop which it is not able to purchase from it makes a small decrement to the shops balance. This is set to a very low value generally, and is intended to make it difficult for shops to trap a number of shoppers who do not need their goods.

The Shoppers

Throughout this study we are not really defining the economics of the shopper as such. A shopper has an infinite purse, so can go on purchasing as long as there are shops which can supply its needs. Our main interest in the shoppers is in how they decide what they want and how they find it. The shoppers navigate the city by looking at the world in front of them and in the direction of their target (if they have one), and choosing either the longest line of sight or that which best matches their target. The distance seen is weighted by the colour of the intervening space but is bounded either by the edge of world, a shop or another shopper.

We have implemented two different methods for shoppers to know what items they want, which we call the "weighted random" and the "multi-need" methods. Each time a shopper makes a purchase their 'need' for another item is reset. The weighted random method makes sure that there are more 'cheap' items needed than expensive ones, since the shops selling cheap goods must attract more shoppers to achieve the same increment to their balance. When a shopper makes a purchase the algorithm ensures that the next purchase is more likely to be cheap than expensive.

The multi-need method is an attempt to make something more realistic, where the shopper is given a list of needs, each of which will go critical within a different time frame, but with cheaper needs going critical at a more frequent rate. At each iteration the shopper checks its needs and if one is getting close to becoming critical and something which matches this need is visible, this becomes the current target, otherwise it waits until the need is more critical before setting a new target and trying to satisfy the new target. Each shopper has a different time profile for all its needs. These shoppers are endlessly distracted by items on offer in their vicinity; the idea was that they are like people in that they may want a rest or a cup of coffee or a meal at regular intervals, no matter what else they're doing so the city should provide these.

We will also provide results on two different methods on finding their targets, one without knowledge of the city and the other with "partial knowledge" of the city. With "partial knowledge", the process works as follows: when a shopper makes a purchase after the shopper gets a new taste, the shopper gets a "global view" of all the shops, their location and their price. It is then able to make a decision on its next target based on price and distance. If the shop goes bust before the shopper gets to it the shopper loses the target and reverts to a "no knowledge" state, for this reason we call it "partial knowledge". With the multi need examples the knowledge is accessed when the target changes.

The City

This then is the whole system, the shape and size of which is bounded by a rectangular 'window' on the screen. The accumulated trace of the agents is shown by the relative darkness of the background, the shops range from blue (cheap) to red (expensive), the shoppers are represented as small black dots of one pixel in size. The front of the shops is shown by a line, in the same way when a shopper purchases from a shop a line flicks out to indicate this.

How Shoppers Become Shops

A shopper can become a shop when a space is available, the shopper has purchased a number of items (between 0 and 10 in these experiments) and the proportion of shops is below a pre set level. In choosing the best place a shopper will survey its surroundings looking for the space which has the lowest shopper trail (the lowest footfall record) in vertical and horizontal directions up to a distance of two shop sizes away. It also tries to avoid all the other shops and the frontage of the shops as defined by the line in front of the shop. The new shop sets its own frontage by a line from where it became a shop to the front left corner of the coloured square as viewed from the shops point of view.

The Experiments

The matrix of experiments we are attempting to run are as follows

Multiple (multi–need)	v	Single item (weighted random) shopping lists
Knowledge	v	No knowledge
Very high tax	v	High tax and low tax
High distance cost	v	Low distance cost

We are aiming to test every combination of the above, at present however we have run all the combinations with low tax and have yet to run high tax combinations with knowledge.

Evaluation

To evaluate the results we will be trying to derive some measures of navigability from statistics accumulated during the simulation. In this case we are looking at the "average balance" of all the shops, the number of purchases made, the number of shopper steps between purchases, as well as how stable the street pattern is. The last of these is a qualative assessment made on the basis of looking at the animation of the settlement's development over time as we look at how well defined the streets have become as defined by the shopper trails in the bitmaps showing the shops and the agent trails. The first three are things we can easily measure without complex evaluation.

We also derive a value for the overall "diversity" of the city, by looking the mean and the deviation from this mean for the different catogaries of shop types, this should really be weighted by the shop life to check that the diversity is actually stable, unfortunately we can't collect these values at present. The values range from 0 to 255 for the average, so 128 is in the middle the spread is then the deviation from this, if its 0 all items are the same as the average, if its 128 all items are at the limits above and below the average. If the average was 128 and spread was 128 all the values would be 0 or 255. So reasonable values are around 64. This should be regarded as a very rough measure as its not time weighted. In this paper we shall limit the catogaries of items to those which are "cheap", having a low purchase value, and those that are "expensive", having a high purchase value.

The factors which indicate the "navigability" of the system are as follows:

- If the average balance is high it means that a lot of purchases are being made, which implies that the agents are finding the shops easily;
- If the number of shopper steps between purchases are low then it implies also that the shoppers are finding the shops quickly. The shopper steps are only evaluated as the period between two purchases to avoid the situation where a shopper becomes a shop or vice versa;
- Lastly, we can look at the actual number of purchases made over a time period.

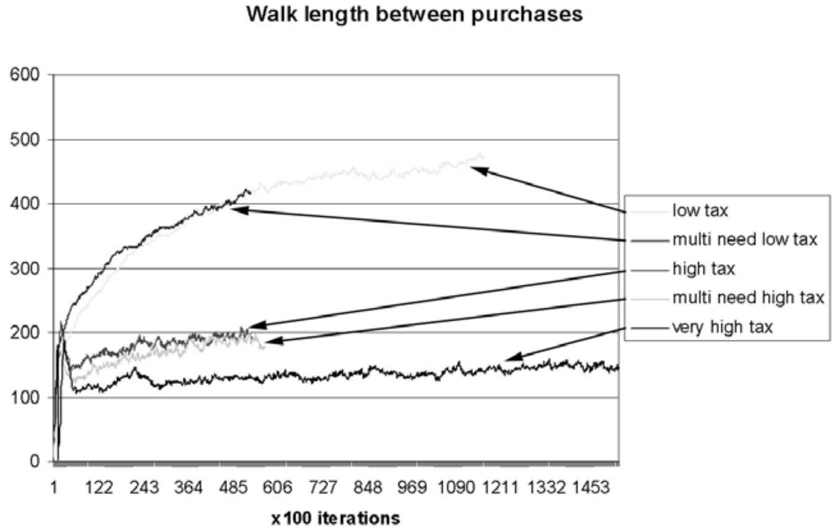
We do not provide statistics at present to show the grouping or otherwise of different kinds of shops, as this would need a parrellel catogorization of type of shop as well as a "price" of shop. This gets very confusing to visualize. However we shall be looking at the distribution of like "priced" shops in the evolving patterns.

Results

In the first two graphs we consider the different effects the economic landscape can have on walk length between purchases (figure 1) and shop balance (figure 2), using the two methods of assigning targets for the agents (random weighted v. multi-need) and under different 'flat' tax regimes. In the second two graphs we look at the effect of giving shoppers knowledge of shop location so that they can set their targets according to the cost of distance as compared to the precise nature of the shoppers need and how closely the item matches this, on walk length between purchases (figure 3) and shop balance (figure 4). Finally, we compare the total number of purchases for both sets of results as well as the average purchase price and the deviation from the average.

Figure 1:

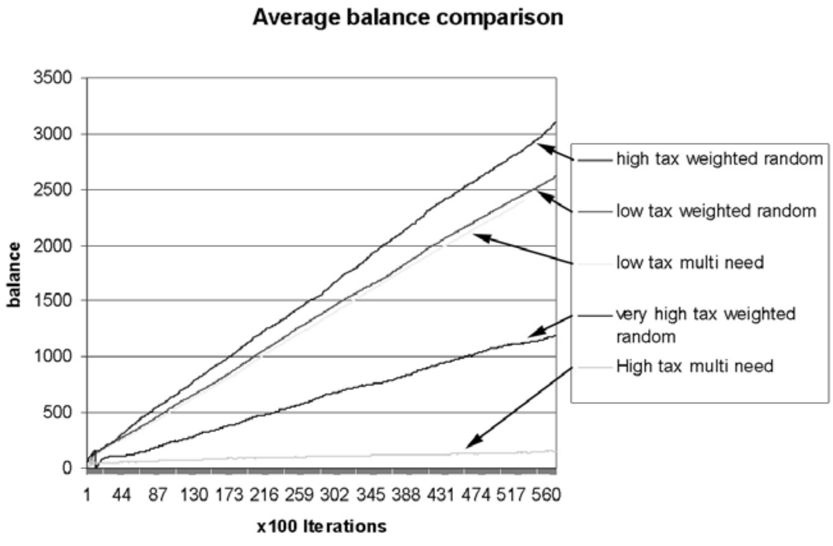
Walk length between purchases as these evolve over time for different tax regimes and single and multi-item lists



Under a low tax regime figure 1 shows how walk length between purchases grows to a maximum and smoothes off. Under high tax regimes, following an initial peak and trough the walk length stabilises at about half that of the low tax. Under the very high tax regime there is a further reduction in walk length. This suggests that the effect of a significant flat tax rate is to force a more optimal layout from the point of view of shopper efficiency.

Figure 2:

Shop balance as this evolves over time under different tax regimes and for single and multi-item lists



This shows the relative "health" of the shops under different tax regimes, using the multi need and the weighted random methods for shoppers to choose their targets. The low tax situations for weighted random and multi-need are very similar, however under a higher tax there is a divergence, where even under a very high tax the weighted random maintains higher balance than the multi–need high tax run.

All the following use the low tax regime used above. We place the "low tax weighted random" into the following graphs so that we can see the difference knowledge makes to the efficency of the system ie it gives us some comparison.

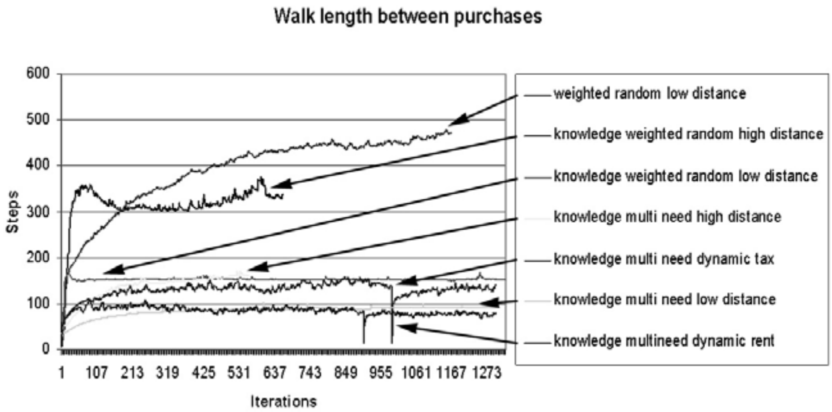


Figure 3:

Walk length between purchases for shoppers with knowledge

All the walk lengths seem to settle more or less to an equilibrium, it is noticeable that most variation occurs at the start of the run, all the runs with knowledge get lower distances walked than those with knowledge, the lowest walks are found in the dynamic rent run.

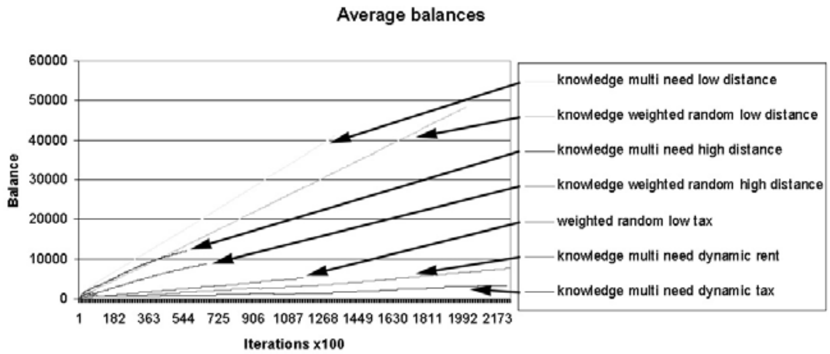


Figure 4:

Average balances for shops for shoppers with knowledge

It is seen that the "weighted random low tax" from the first pair of graphs has one of the lowest values in this graph, with only the dynamic tax and rent at a lower value. All the average balance graphs ascend at a more or less even gradient so we shall take a sample at 53300 iterations to be the representative value. Similarly the average walk length settles to a kind of equilibrium so we will take an average from the period around 53300 iterations. The following Table 1 gives a summary for first 53300 iterations for the first set of runs of agents without knowledge.

Table 1:

Summary of sytem measures for agents without 'knowledge'

	Purchases	Ave spread	Ave balance	Walk
Very high tax, weighted random	6693	149,61	1120	129
High tax, weighted random	5649	156,57	2845	201
High tax multineed	4195	125,61	144	190
Low tax , weighted random	3109	129,54	2392	419
Low tax multineed	3027	112,50	2407	417

We can see form Table 1 that the highest tax gives us the shortest average walk and the greatest number of purchases. The average balance for the "High Tax weighted random" run is unbelievable and might be due to some accident in this particular run, a particular area of shops finding an optimum and forcing up the overall average, this needs further investigation. Overall the upwards trend in Purchases as the tax increases and the average walk length are not affected by the "High Tax weighted random" average balance anomaly.

The second set of runs gave the shoppers knowledge though in a low tax situation and with a high and low cost for distance.

Table 2:

System measures for shoppers with knowledge

	Purchases	Ave spread	Ave balance	Walk
Knowledge, w random, high dist	3495	144,58	7521	333
Knowledge, w random, low dist	7100	140,61	12900	157
Knowledge, multi-need, high dist	6904	97,47	11651	151
Knowledge, multi-need, low dist	12694	94,48	16977	81
Weighted random, low distance	3109	111, 51	2462	430
Multi-need, dynamic rent, low dist	6731	123,58	1529	80
Multi-need, dynamic tax, low dist	6269	125,58	847	146

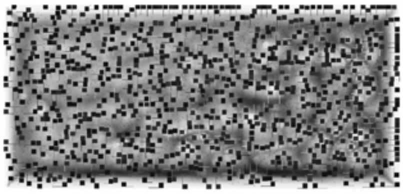
We can see from Table 2 that the highest number of purchases occurs with the multi need at low cost of distance, this also has one of the shortest walks. The relative health of the dynamic rent run is also interesting, where a low walk and reasonable number of purchases is only dampened by a low average balance. It is also noticable here that a higher weighting for distance seems to reduce the number of purchases, contrary to expectations. It is possible that the distance weighting was set too high, so perhaps more experiments are needed to investigate this further.

The multi-need method gives us the highest number of purchases, and when you give knowledge to the shopper, the handicap that was "multi need" in the first set of runs becomes an asset. In other words, when shoppers have multiple needs to trade off against their current location they can take advantage of this fact to behave more efficiently.

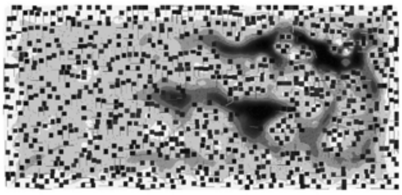
The theoretical maximum number of purchases would assume a shopper makes a purchase every 10 steps (the shop frontage length). With 3000 agents and with the proportion of shops at 0.3 this gives us 2100 shoppers and 900 shops. The maximum number of purchases per iteration would therefore be 210. In our runs the highest number of purchases per iteration was 86, with an average of 22 for the whole run. It is possible then if an area starts to behave in an optimal fashion (with shoppers making purchases every 10 steps) that this could skew the results massively. We have yet to treat the issue of sampling and robustness rigorously. It is possible that these runs should be repeated several times for each setting, with a time based random seed to avoid repeating runs, before we can really rely on the numeric results, however although individual shops may be replaced under certain parameter settings, visually at least it is possible to see when a settlement morphology becomes relatively stable. These stable morphologies show a surprising range of types.

In figure 5 image a) shows what happens when the economics of the system do not allow a stable structure of shops to appear, this is a worst case example and is shown for reference. In all the images except d) and e) (the high and very high tax examples) the shoppers have knowledge of where the shops are. Also the only example showing the multi-need method of choosing where to go is g). All the others use the weighted random method of choosing their next target.

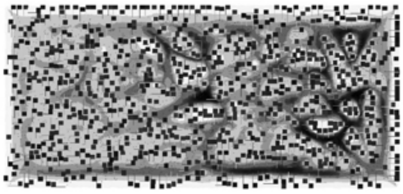
The pictures show several different forms of order. We have a grid like pattern in d) and e) where our shoppers have no knowledge and more diagonal patterns in f) and g) where the shoppers have knowledge. b) and c) could be characterised as "blobby" with several separate centres which aren't really connected.



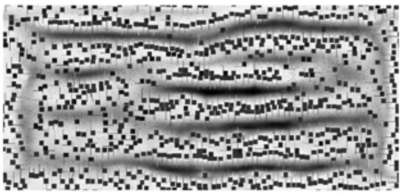
a) This shows a failure to establish any discernable pattern



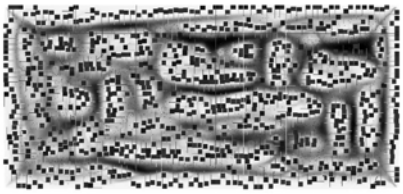
b) Here shops are taxed dependent on the number of passing shoppers. Av. balance 1529, Av. walk 80. Purchases 6731



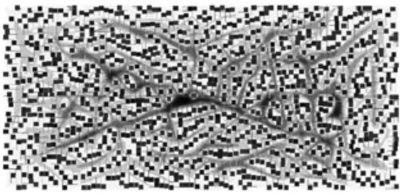
c) All shops are taxed equally, though dynamically. Av. balance 847, Av. walk 146. Purchases 6929



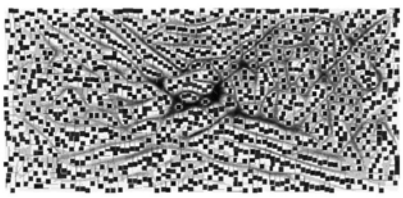
d) This shows the effect of very high taxation. The shops have formed themselves into rows. Av. balance 1120, Av. walk 129. Purchases 6693



e) This is a high tax example, the diversity is good and a grid has begun to form. Av. balance 2845, Av. walk 201. Purchases 5649



f) The low tax example, has good diversity, with some structure. Av. balance 12900, Av. walk 157. Purchases 7100



g) The multi-need low tax example, has more"blue" (cheap) shops than the random weighted above. Av. balance 16977, Av. walk 81. Purchases 12694

Figure 5:

Visualisations of settlement morphology with shops and darker traces showing shopping agent flow density

The resulting tax level from the dynamic tax was 1.4 which gives us the maximum tax at which the knowledge with multi-need will operate, so if you repeat g) with a tax set at 1.4 then the pattern in g) should change to that in c). It is possible then that the high values found in g) are temporary, as the pressure towards optimisation would probably lead eventually to the pattern and purchases of c). In the "no knowledge" runs higher tax led to a greater number of purchases, were you to add knowledge to the situation the opposite might be true, with an increase in taxes leading to less purchases.

Discussion

We have found that we can alter the shape and layout of our virtual city by altering both the nature of the inhabitants of the city, in terms of the definition of their motivations and cognition, and by applying different tax policies. We would suggest that the precise spatial form which evolves under the simulation is driven by how the shops arrangement with respect to each other affects the shopper fulfilling their motivation given their perceptions of their environment are constrained by forward facing vision and their level of knowledge of shop location. It is this set of factors that defines how easy it is for the shopper to get from shop to shop without interruption. It would seem that policies which punish shops which are not visited, are highly effective at driving up the number of purchases while forcing the shops into neat rows, and so aligning the visual field to create long distance views. If we were describing the space as a communicable field we would say we were removing clogging material from the passageways and straightening out the routes to improve visibility. How this translates to the improvements found where the shoppers have knowledge of the area and have the more irrational method of choosing targets would require further investigation. It is clear that knowledge reduces the importance of the configuration for delivering agents to the shops as they know their way anyway. However the less changes in direction needed to avoid shops which are in the way, the quicker the agent should find its way to the next shop, and at a population level we would expect to see greater efficiencies emerge. In the dynamic tax situation we don't seem to see any optimisation of this kind occurring, though effectively we are running the knowledge multi-need simulation at a higher tax rate, so unless there is something in the dynamic system which stops a good layout from appearing we need to consider other mechanisms.

We were hoping the dynamic tax would provide a periodic jolt to the system as happens in simulated annealing, to shift the model out of local optima, but it seems that the way it is applied here leads to a degradation of the performance of the system. However the variable rent seems to provide a pressure to reduce journey lengths. This might be usefully combined with some other factor, though in reality having shorter walk lengths with less purchases doesn't really seem possible, unless all the shoppers are turning into shops and back again without moving.

It is noticeable too, that when we raise taxes and obtain the more street like patterns these are largely constructed from the more expensive shops. This is because the more expensive shops require less visits to remain solvent. Long term cheap shops will only be found at locations where a lot of shoppers pass, thus over long time scales a form of spatial sorting can occur. In practice, the cheaper shops tend to have shorter lives amongst a developing structure of more expensive shops. This is easier to see when the evolution is shown as a speeded up movie.

Having seen shapes which result from some economic pressure it would be interesting to see whether shoppers constrained to street plans with variable tax rates would be able to maintain a total occupation of all the available space and what the resulting tax rate would be. Also there would be an interesting comparison between the different kinds of shoppers. Would the shoppers without knowledge perform best in grids, and the shoppers with knowledge in the more diagonal to amorphous patterns seen in c) and g)? We are aware that the matrix of behaviours ascribed to the shoppers and the form of the economy presented in these simulations so far are extremely limited, however we felt that given the constraints of computers, the visual behaviour and the knowledge were most computable for real time simulations.

Conclusion

The findings of this set of experiments, although not entirely conclusive, suggest an interesting possibility. By coupling a cognitive agent – cognitive in that they have desires, intentions and beliefs about the world derived through their perceptions – to an economic process, it seems as though order emerges in the environment in the form of deformed grid like open space structures, but only under the conditions where the agent has no explicit global knowledge of the environment. Where global knowledge is given to the agent, order in the environment considered as globalising linear arrangements of space, appears to be reduced, however this is replaced by a 'memory' of the original central entry point for the agents where the early evolution of the settlement persists in the form of a centralised structure and radial routes. These are two properties of different growth periods in European urban development – a centralised and radial structure characterises the early stages of urban development when a city is small enough to be 'knowable' in its entirety, however as it grows it is often surrounded at a later period by larger tracts of grid-like development. Others have of course shown simulations of economic urban forms (Allen & Sanglier, 1978; 1981; Prigogine & Stengers, 1984), and these also showed the emergence of centralised patterns of density, however these simulations were not defined at the level of open spaces and building forms that might characterise street systems. Similarly, others have simulated urban growth processes through aggregation of buildings according to various kinds of joining rules (Hillier & Hanson, 1984; Erickson & Lloyd Jones, 1996), however these simulations assume rule sets that apply to the morphology itself like building regulations, rather than processes that apply constraints to the economic success or failure of premises themselves through their effects on mobile inhabitants.

It would of course be farfetched to suggest that simulations of the simplicity and level of abstraction presented here mirror the processes that have developed real centralised and grid-like European cities, however the notion that coupling between knowledge and economic drives, under the constraints on information derived by visual perception of mobile agents, can give rise to these kind of recognisable morphological outcomes is persuasive. We have yet to investigate the coupling of these effects with aggregation or dispersion of shops selling similar goods, or more complex representations of knowledge and dynamic learning on the part of agents, and these will form the next phase of our research.

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